PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 9 AUGUST 15, 1923 Number 8

THE VIBRATION OF AIR IN TUBES CAPPED AT BOTH ENDS*

By Carl Barus

Department of Physics, Brown University

Communicated June 15, 1923

1. *H-tubes.*—The arrangement is shown diagrammatically in figure 1 (inset), where tAt' is the tube, $2R = 2$ cm. in diameter. A is extensible, but like the whole region air tight. T and T' are two common telephones by which the air columns are actuated, the switch C allowing of reversal of the current in T relatively to T'. The junction of t with the mouthpiece of T and of t' with T', are rigidly cemented. The far ends of t and t' are closed with perforated corks, through which the conical pin-hole probes s (salient) and r (reëntrant) are inserted. One of them (here s) communicates by a short end of rubber tubing with a shank of the interferometer U-tube. Curiously enough it makes little difference whether both s and r are joined with the corresponding shanks, or whether one, r or s, is open to the atmosphere.

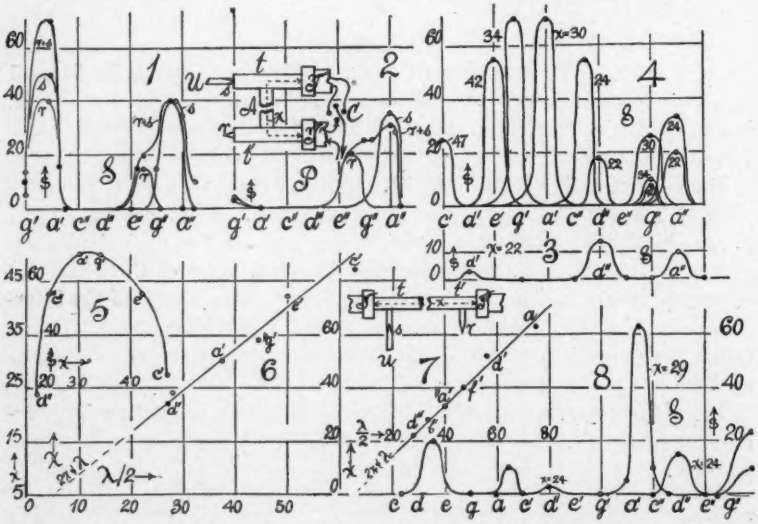
The telephones are energized by a small induction coil with a large resistance in circuit. A controllable break-circuit (electric siren) is in the primary.

In figures 1, 2, s denotes the fringe displacement of the interferometer U-tube and is proportional to the acoustic pressure generated at r, s (inset). S (sequence) shows that the telephone plates are vibrating in opposed phases, P that their phase is the same. The graph thus records the observed nodal intensities at the different frequencies (pitch) given by the abscissa.

To obtain the r curve, figure 1, the s pin-hole was plugged, and vice versa for the s curve; when both pin holes are used, the effect is here not quite summational at the g' to a' resonance; but it frequently is so. The upper separate resonances e''–g'', g''–a'' differ in pitch; consequently when r and s are used together $(r + s)$ there is an interesting multiresonance, or blend. One may note that for these wide tubes, $R = 1$ cm., a closed region shows acoustic pressure (curves r, s), even with a single pin-hole within.

In figure 1 the telephone plates are opposed in phase (sequence S), while figure 2 shows the corresponding results with the plates vibrating in phase (P). The g'–a' resonance has vanished, whereas the e''–g'', g''–a'' maxima retain their character except as to intensity. Consequently the g'–a' maximum must be associated with the axial tube length x (figure 1, inset) from plate to plate of the telephone. The upper (fixed) maximum will be found largely referable to the telephone.

Figure 4 shows what happens when the cross tube A (figure 1, inset) is elongated so that x = 22, 24, 30, 34, 42, 47 cm., in succession. The g''–a'' maxima remain relatively stationary, whereas the low pitch maximum moves from d'' to c'. In this march it passes an optimum of in-



tensity (x = 30–34), more fully exhibited in figure 5, which contains the relation of x and s. In figure 8 a more extended survey in pitch, for x = 24 and 29 cm. and from c to c''', is reproduced. In figure 3 the pin-holes r and s communicate with the atmosphere, while the U-tube is joined to the middle of A without a pin-hole. The registry of acoustic pressure is definite but low in intensity.

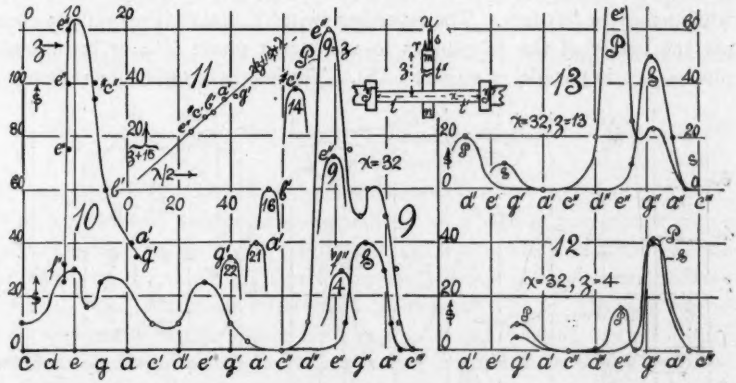
If the maxima of figure 4 be used to construct x in terms of the free wave length λ, the graph in figure 6 results, which is as nearly linear as anything else and equivalent to $2x = 0.79\lambda$. Thus there is a marked frictional effect[1] but no special frequency effect, such as would be anticipated theoretically.

Straight Tubes.—The adjustment is given diagrammatically in the inset, figure 7, r and s being the pin-hole probes very near the telephones

T and T'. The U-gauge is joined to s. The results, as a whole, were very similar to the preceding. I will only exhibit the relation of tube length x and λ which has the form $2(x + a) = b\lambda$, or $2(x + 4) = 0.92\lambda$, where a is the effect of the telephone mouthpiece expressed in length of tube. The coefficient here is considerably larger, a result to be anticipated since these tubes were both straight and smoother. Again there is no evidence of a special frequency effect.

Transverse Tubes.—The adjustment is shown in the inset, figure 13, T, T' being the telephone cemented to the straight tubes t, t', across the middle of which is the transverse tube mn of effective length z and carrying the pin-hole probes r, s at the m end, s being joined to the U-tube.

It is clear that the tube will now respond effectively when the telephone plates vibrate in phase (P). The S adjustment evokes the fixed telephone



resonance only. Examples of fringe deflection s in terms of pitch are given in figures 12 and 13, for $x = 32$ and $z = 4$ and 13 cm., the movable note being near e'' and tending toward an optimum. This is more fully shown in figure 9, in which the successive values ($z = 4$, 9, 14, 16, 21, 22 cm.) are attached to the maxima. The graph is fully worked out for $z = 9$ cm. for the P vibration. In the S vibration of the telephone plates, only the fixed g'' appears distinctly. Figure 10 is the (s, z) graph, indicating the very rapid tendency toward and from the optimum (max. s) as tube length increases. Finally in figure 11, the result $z + a = z + 15 = 0.88\lambda$ is exhibited. Thus since $a = x/2$ nearly, the vibration of the air column is from both T and T' plates toward m and return. The coefficient $b = 0.88$ agrees as closely with the preceding $b = 0.92$, as the ear can detect chromatically varying pitch.

* Advance note from a Report to the Carnegie Institution, Washington, D. C.

[1] Cf. Rayleigh's "Sound," Chapter IX, discussing the equations of Helmholtz and others.

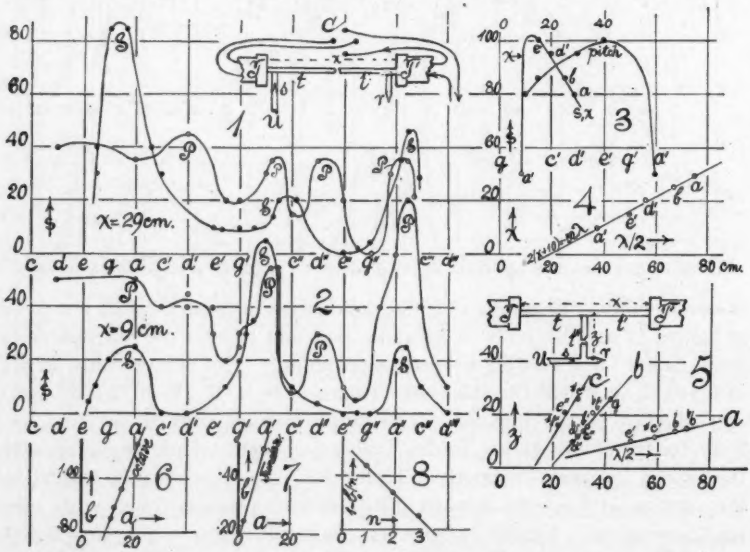
THE VIBRATION OF THE AIR FILAMENT IN QUILL TUBES CAPPED AT BOTH ENDS*

By Carl Barus

Department of Physics, Brown University

Communicated June 25, 1923

1. *Straight tubes.*—The plan of the experiments is similar to the preceding group for wide tubes. In figure 1 (inset) t, t' is the quill tube 0.35 cm. in diameter, actuated by the paired telephones T and T' at the ends and with salient (s) and reëntrant (r) pin-hole probes near the telephone plates. One shank of the interferometer U-gauge communicates with s, while r may be left free. The reversing switch C makes it possible to compel the plates of the telephones to vibrate in phase (P), or in opposed phases, i.e., in parallel or sequence (S). The length of quill tube is denoted



by x (if transverse by z) and the fringe displacement proportional to acoustic pressure by s. The telephones and the variable break circuit of a small inductor constitute an electric siren.

The graphs, figures 1 and 2, for tube lengths $x = 9$ cm. (the smallest available) and $x = 29$ cm. (the largest), for the like phase adjustment P at the two ends, are of the same character with a difference in intensity distribution. In the four foot octave, s is nearly constant or the resonance

is continuous. In view of the end position of the pin holes, it is difficult to detect a maximum moving with x. In the S adjustment, however, besides the fixed nodes, the principal maximum S drops about an octave (a' to g) as x increases from 9 to 29 cm. This is the fundamental quill tube note.

In figure 3 the intermediate cases are given, the resonance displacement being summarized in relation to tube length x and also to pitch. The usual optimum, here between 10 and 15 cm., is pronounced even in this case of straight tubes.

Figure 4 finally shows the relation of tube length x and free wave-length λ and may be reproduced by $2(x + a) = b\lambda$, where the accessory tube length a = 10 cm. and the coefficient $b = 0.51$. As in the earlier paper, no special effect of frequency, n, is apparent within the octave examined, though the frictional effect is very marked. If we write the equation of Helmholtz and Rayleigh in terms of wave-length (λ free, λ' imprisoned) it may be put (viscosity η, density ρ, frequency n, radius R)

$$\frac{\lambda-\lambda'}{\lambda} = \frac{\lambda/2 - (x+a)}{\lambda/2} = \sqrt{\eta/R^2 2n\rho}$$

so that in the absence of a special frequency effect $R(1-b)$ = const. Thus for the two diameters examined, $2R$ = 2 cm. in the former paper and $2R = 0.35$ cm. here, the coefficients are $1-b = 0.08$ and 0.49. While $2R$ has been decreased 5.7 times, the coefficient $1-b$ has increased about 6 times. But the small differential coefficient $1-b$ = 0.08 is of course not trustworthy to one or two units in the second decimal, so that all the relations could quite well be reconstructed in accordance with $(1-b)R$ = const., throughout.

2. *Transverse tubes.*—The case of transverse tubes of total length z (figure 5, inset), of which I will only give the graphs for 3 harmonics in figure 5, presents much greater complications. The length of salient (s) and reëntrant (r) pin-hole probe (s joined to the shank U of the interferometer U-gauge) are not negligible here but must be included in z. The telephone plates vibrating in phase (P) evoke the successive resonances, but the curves corresponding to figures 1 and 2 must here be omitted.

The graph b was first worked out. In connection with it, the graph c was detected. As this is a scant fifth above graph b, the fundamental graph a was suspected and subsequently found. Its intensities (s, not shown) were in fact the largest, b intermediate and c small. So far as the data go, the following equations suffice to reproduce them, using $2(z + a) = b\lambda$:

frequency ratio:

n = 1	$z + 2.5$ =	.25(λ/2)	λ' =	0.25λ
2	$z + 22$ =	1.13(λ/2)	λ' =	0.56λ
3	$z + 22$ =	1.65(λ/2)	λ' =	0.55λ

where $(z + a)/n = \lambda'$, the semi wave-length under friction. The last two coefficients might be taken to be of the same order as $b = .52$ in the preceding section for straight tubes; but the coefficient of the fundamental $b = .25$ is out of accord with this. It is tempting to refer the discrepancy to the constriction in t'', of the vibration in t and t', the sectional area being halved. In such a case, however, the same result should appear for $n = 2$ and $n = 3$. The theoretical equation $(1-b/n)\sqrt{n} = \text{const.}$ fails equally to give a consistent explanation.

The most puzzling feature is the interdependent variation of a and b in successive experiments and due to incidental causes not detected. Thus in case of the octave

$a =$	12	13	16	22
$b =$	.85	.89	.96	1.13

were found at different times. Similarly for the fundamental $a = 2.5$, $b = .25$; $a = 10$, $b = .51$ are given above. Figures 6 and 7 show that these relations are grouped consistently and do not differ much for the fundamental and the octave. Hence the value of $b = b_0$ for $a = 0$ is determinable and the reduction for the fifth may be made with the same coefficient as for the octave; i.e., $b = b_0 + \beta a$.

Thus $x + a = (b_0 + \beta a)\lambda/2$, for $a = 0$ becomes $x = b_0\lambda/2$, while $x = n\lambda'/2$ so that $\lambda' = (b_0/n)\lambda$ and finally

$$(\lambda - \lambda')/\lambda = 1 - b_0/n$$

Numerically this amounts to $1-b_0/n = .94-n/10$ or $b_0 = .06n + n^2/10$; or again,

$n = 1$	$b_0 = .16$ (observed)	$b_0 = .16$ (computed)
2	.52	.52
3	1.05	1.08

Figure 8 indicates the proportionality of $1-b_0/n$ with n. Insofar as this general method of contrasting harmonics as a whole and of eliminating the a,b dependence is valid, a special frequency effect in b_0 cannot be rejected; but it does not follow the theoretical equation.

* Advance note from a request to the Carnegie Institution, Washington, D. C.

CORRELATION BETWEEN PHYSICAL AND MEDICAL FINDINGS ON NORMAL EARS

BY JOHN P. MINTON AND J. GORDON WILSON, M. D.*

RADIO CORPORATION OF AMERICA AND NORTHWESTERN UNIVERSITY MEDICAL SCHOOL

Communicated June 15, 1923

Introduction.—During the past three or four years much time has been given to the study of the sensitiveness of normal ears for tones of various pitches and also to a correlation of the physical and medical findings in cases of slight deviation from normal. The present paper is a contribution to existing data (see references) published by the writers and others in that curves for individual ears are presented and are discussed from the viewpoint of both medical and physical observations. The data for the curves were taken as described in an earlier paper[7b] by one of the writers. In the present work, however, the telephone receivers had been calibrated so as to give the minimum audible pressure in dynes acting on the ear drum instead of giving the vibrational energy of the receiver diaphragm at the threshold of audition. The assumption of the doubtful equivalence of the drum membrane and a metallic diaphragm is present in this work, as in the results of other writers. This method, however, was adopted in order that the results published herewith may be compared with those of other writers.

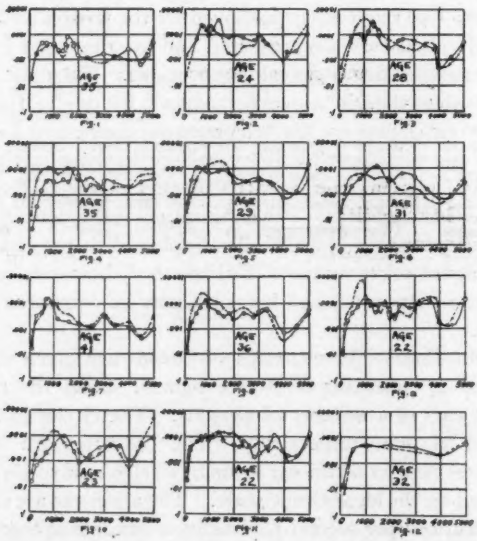
Experimental Data—Three groups of curves are given in the present paper. Group I, consisting of twelve figures, shows the curves for the right and left ears of a number of persons. The circles and dots are for the right and left ears respectively. The ordinates are the root mean square pressures exerted on the ear drum by a telephone receiver diaphragm, and are plotted on the logarithmic scale. The abscissae are the frequencies in double vibrations per second plotted on a linear frequency scale. The ages of the persons are shown in the figures.

The otological findings for these persons whose curves are given in figures 1 to 12 are as follows: Each ear in this group of twelve figures gave no history of pain, tinnitus, discharge or vertigo. The examination showed normal external canals, normal drum membranes, and no disease in the nose or pharynx. The whisper test was normal. The Rinne, the Weber and the bone conduction tests were normal. The perception and duration of the forks 64 d.v., 512 d.v. and 1024 d.v., and of the Galton whistle were normal. The only exception was for figure 10 which showed in the right ear a marked break in the light reflex and gave the Weber test to the right but no other abnormality.

For the purpose of serving as a check on the first group of tests another receiver was used for the second group of tests. The seventeen curves

shown in group II were taken with this receiver. As in group I, circles and dots are for the right and left ears respectively. Crosses, wherever used, are for left ears.

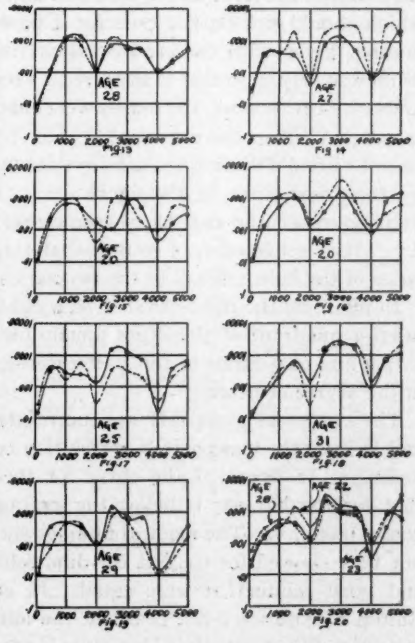
The medical findings for these cases as observed by the otologist are as follows: Each ear in this group gave no history of pain, tinnitus, discharge or vertigo. The examination showed normal external canals, normal, drum membranes, and no disease in the nose or pharynx. The whisper test was normal. The Rinne, the Weber and bone conduction tests were normal. The perception and duration of the forks 64 d.v., 512 d.v., and 1024 d.v. and of the Galton whistle were normal. The following cases showed slight exceptions: Figure 17, the tonsils were enlarged and the



crypts were full of debris. The left drum membrane showed the light reflex broken so that it appeared as a very broad base with nothing at the umbo. The whisper test was diminished. The right ear was normal. Figure 19, drum membranes showed no light reflex, but the otological tests were normal. Figure 20, the right ear showed a patch of calcification in the anterior inferior quandrant near the periphery. The otological tests were normal.

In group III are found curves on ears which were examined for normal ones and which were found to be more or less abnormal by both the physical and medical findings. A detailed discussion of them will be found below. The curves in the first five figures of this group were taken with No. 1 receiver and those in the last figure of the group were taken with No. 2 receiver.

Discussion of the Experimental Data.—All of the curves are alike in that they show regions of maxima of sensitivity, but these regions are often different for the two ears of the same person and these regions vary widely among the different people. For perfectly normal ears, the maxima of sensitiveness occur as low as 500 d.v. for some ears and again they are not reached until the frequency is as high as perhaps 3000 d.v. All the curves showed an increase in sensitivity at 5000 d.v. It is believed, however, that this was due to the receiver, in as much as their natural periods were above 5000 d.v. and perhaps the calibration did not entirely correct for this. Then, again this rise may probably be due to resonance in the ear cavity when closed by the receiver, although the calibration would at least partially correct for such resonance.



When the average curves are plotted, the maxima and minima are more or less flattened out. Figure 27 shows the average for the two receivers. The receiver tuned to a natural period 5620 d.v. gives a satisfactory average curve. The average curve for the other receiver has depressions at 2000 d.v. and 4000 d.v. which we could not account for. The agreement between the two receivers is perhaps not satisfactory in the region of 3000 d.v. Of course, different sets of ears were tested with the two receivers and for this reason the agreement may perhaps on the whole be considered satisfactory.

If a comparison of these average curves is made with that given by Fletcher and Wegel,[2] it will be observed that the average sensitiveness is somewhat superior to that obtained by the authors referred to. This is in agreement with the latest results published by Kranz.[5] The average ages of the people tested may be different in the two cases, a difference would also be caused by the selection of the ears to be classed as normal, and furthermore, the tones used by the writers were intermittent, while those used by Fletcher and Wegel were continuous during their tests at each frequency. The threshold value in the former case is lower than in

the latter. Perhaps this accounts for the lower average threshold pressure values reported herewith than reported by Fletcher and Wegel.

The correlation of the medical observations with the curves of sensitivity proves of much importance. In no case do the findings obtained by the otologist on the one hand and the physicist on the other diasgree to any noticeable extent. In figure 10 the right ear is lower in sensitivity than the left one below 1200 d.v. and the light reflex observed by the otologist showed the right drum membrane somewhat abnormal.

In figure 17 the degree of hearing for the left ear is considerably lower than that of the right above 1800 d.v. and it is also lower than the average which should not be the case for a person 25 years old. The medical findings agree with these observations in that the light reflex from the drum was broken so that it appeared a very broad base with nothing at the umbo. Furthermore, the tonsils were enlarged and the crypts full of debris as pointed out in the preceding section. These facts show that one can expect to find a diminished sensitiveness at the higher frequency as shown by the lower curve in the figure.

In figure 19 the curves are somewhat low above 3000 d.v. or 3500 d.v. It is not known, of course, that this fact is associated with the absence of the light reflexes in the two ears as stated in the medical findings.

In figure 20 the right ear showed a patch of calcification in the anterior inferior quadrant of the drum membrane near the periphery but the location and magnitude of this patch were such as to produce no depression in the degree of hearing.

The curves in group III are more interesting than those in groups I and II from the viewpoint of correlation between the medical and physical findings. In figure 21 the curve for the right ear is a normal one but that for the left ear is below the average at practically all frequencies above 1000 d.v. The medical findings showed the right ear to be normal but they showed for the left ear diminished air conduction until both air and bone conduction were equal. In addition, the high notes of the Galton whistle were not heard in the left ear. The patient has had for some time slight attacks of loss in balance and vertigo on rapid movement of the head. In addition, the patient had some infected teeth which had not been removed up to the time of the tests. The agreement between the two kinds of aural examinations is satisfactory.

In figure 22 are given three curves. One (shown by the crosses) is for the left ear of a person 37 years old, the other two are for the right and left ears of a person 29 years old. At the time of the tests, the latter person had a very bad cold in the head and a week previous to the examination had a slight pain in the right ear, but none in the left. The right drum membrane was normal except a punctuate hemorrhage at the umbo. The left drum membrane had a slight reflex present only at the base. The

Weber test was to the left. The curves indicate normal hearing and these temporary conditions observed by the otologist were of such a nature as to not affect the degree of hearing.

With regard to the curve for the left ear of the person 37 years old, it is seen that the sensitivity is unusually large from 500 d.v. to 2000 d.v. but is seriously depressed above the latter frequency. The medical findings are also interesting. In July 1921 in the left ear there was present acute otitis media with vertigo accompanying an acute coryza. In addition, the patient had a pronounced case of diplacusis for two or three weeks after the attack. This ear showed calcareous deposit in the membrane anterior to the malleus. The Rinne test was positive and the high notes and the whisper were greatly diminished. So that, both the middle and internal ears were affected and the depression for all the high notes suggest a serious internal ear involvement.

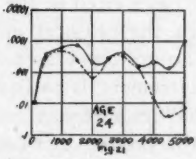


Fig. 21

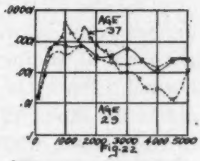


Fig. 22

Figure 23 shows curves for a person whose drum membranes have a very diffused light reflex showing that they were not in normal position and that they were less than normal thickness. The tonsils were considerably enlarged and the whisper test was not satisfactory. These findings agree with the fact that the curves are much below the average, particularly the ones for the left ear.

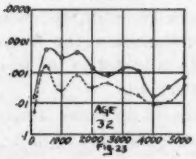


Fig. 23

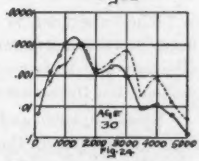


Fig. 24

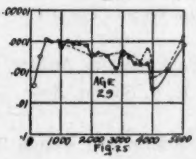


Fig. 25

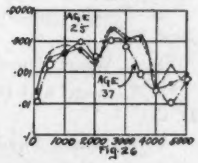


Fig. 26

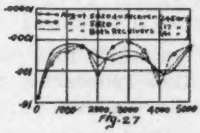


Fig. 27

The persons whose curves are shown in figure 24 had retracted drums, a slight sore throat, muco pus in the nose and a diminished whisper test at the time of the examination. The patient's hearing in the two ears was greatly depressed over the higher frequency region.

The curves in figure 25 are given to show the improvement in hearing that occurred for the higher pitches when three infected teeth were removed. The dotted curve was taken a few weeks after the removal of the teeth. The change was quite marked for all the higher frequencies but the infection did not seem to affect the hearing for the tones of low pitches. This probably agrees with otological experience.

Three curves are shown in figure 26. The large circles are for the right

ear of the person whose curve for the left ear was shown in figure 22. The other two curves are for the ears of a person 25 years of age. This person had recurring attacks of pain in both ears and both drum membranes were retracted and thickened. His curves show a marked depression above 4000 d.v. in both ears. One might expect the abnormal drums to cause a depression below normal at the lower frequencies, but this did not show up.

The curve passing through the large circles is much depressed above 3000 d.v. and normal below that pitch. Hence, it is identical with the curve for the left ear shown in figure 22. In both ears the patient had otorrhoea from the age of 19 to 21. The right ear has a slight depression marking a healed perforation in the posterior superior quadrant and a calcareous deposit anterior to the umbo.

In all the fifty-four cases cited in the present paper there are only one or two cases in which the physical measurements of sensitiveness and the medical findings do not agree as to whether an ear is normal or abnormal. Since the agreement is satisfactory, these curves are valuable aid to the otologist and are surely of satisfaction, at least, to the patients in comparison with the present methods of test used by the otologists.

* The experimental work given in the present paper was done while Dr. Minton was a NATIONAL RESEARCH FELLOW at the Ryerson Physical Laboratory, University of Chicago and he is greatly indebted to the members of the department for their kind coöperation throughout the work.

[1] Seashore, Dean and Bunch, (*a*) *Trans. Amer. Otol. Soc.*, **15**, 1920 (37–60); (*b*) *Psychological Monograph, University of Iowa*, No. 8, 1922.

[2] H. Fletcher and R. L. Wegel, These PROCEEDINGS, January, 1922. *Phys. Rev.*, June 1922.

[3] R. L. Wegel, These PROCEEDINGS, August, 1922.

[4] R. L. Wegel and Dr. E. P. Fowler, *Amer. Otol. Soc.*, May meeting in Washington, D. C.

[5] F. W. Kranz, *Physic. Rev.*, p. 384, March 1921, and p. 573, May, 1923.

[6] John P. Minton and Dr. J. Gordan Wilson, *Proc. Inst. Med., Chicago*, 1921, pp. 157–171.

[7] John P. Minton, (*a*) These PROCEEDINGS, August, 1921; (*b*) *Physic. Rev.*, February, 1922; (*c*) These PROCEEDINGS, September, 1922.

THE EINSTEIN EQUATIONS OF THE GRAVITATIONAL FIELD FOR AN ARBITRARY DISTRIBUTION OF MATTER

BY TRACY YERKES THOMAS*

PRINCETON UNIVERSITY

Communicated June 17, 1923

1. Einstein assumes that the trajectories of the motion of a mass particle in a gravitational field are given as solutions of the differential equations

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0 \qquad i, j, k = 1, 2, 3, 4. \tag{1.1}$$

where (x^1, x^2, x^3) denote the position of the particle at the time x^4 and s is a parameter. The Γ's are the Christoffel symbols with respect to a symmetric tensor g_{ij}, i.e.,

$$\Gamma^i_{jk} = \frac{1}{2}g^{i\alpha}\left(\frac{\partial g_{\alpha j}}{\partial x^k} + \frac{\partial g_{\alpha k}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^\alpha}\right)$$

g^{ij} is the cofactor of g_{ij} in the determinant

$$g = \begin{vmatrix} g_{11} & \cdots & g_{14} \\ \vdots & & \\ g_{41} & \cdots & g_{44} \end{vmatrix}$$

divided by g. The quantities g_{ij} are called the components of the gravitational field and are given as solutions of the field equations

$$B_{ij} = 0 \tag{1.2}$$

where $B_{ij} = B^\alpha_{ij\alpha}$, and

$$B^i_{jkl} = \frac{\partial\Gamma^i_{jl}}{\partial x^k} - \frac{\partial\Gamma^i_{jk}}{\partial x^l} + \Gamma^i_{ak}\Gamma^a_{jl} - \Gamma^i_{al}\Gamma^a_{jk}.$$

(1.2) holds at a point of the field at which no matter is present. As is customary I shall consider the tensor g_{ij} to determine a Riemann space and shall consequently speak of the metric structure of this space.

2. The Schwarzschild solution of the field equations gives as the metric structure of the space-time manifold for the case of a stationary gravitating mass, an expression of the form

$$ds^2 = \left(1 - \frac{A}{r}\right)dl^2 - \left[\frac{dr^2}{1 - A/r} + r^2(\sin^2\theta d\varphi^2 + d\theta^2)\right]. \tag{2.1}$$

This metric is referred to a spherical coördinate system in which the mass M is located at the origin. If we denote by K the Newtonian gravitational constant and by c the velocity of light, then

$$A = 2KM/c^2, \qquad l = ct, \qquad t = \text{time}.$$

Since the Newtonian potential V due to the mass M is

$$V = -KM/r, \qquad 2.2$$

the expression (2.1) may be written

$$ds^2 = \left(1 + \frac{2V}{c^2}\right) dl^2 - \left[\frac{dr^2}{1+2V/c^2} + r^2(\sin^2\theta d\varphi^2 + d\theta^2)\right]. \qquad 2.3$$

3. Let us now consider the gravitational field produced by any *stationary* distribution of matter. The Newtonian potential of the field at a point P at which no matter is present is given by

$$V = -K\int \rho/r_\rho . dv$$

where ρ is the density at a point which is a distance r_ρ from P and the integration is over the entire field. The equipotential surfaces are the surfaces

$$V = \text{constant} \qquad 3.1$$

If we denote by n the direction normal to an equipotential surface in which the resultant force acts, then this force is $-\partial V/\partial n$. The potential and the gravitational force at any point P of the gravitational field are the same as though due to a mass C_ρ/K located a distance R from P at a point O on the normal to the equipotential surface at P provided that C_ρ and R have the values

$$C_\rho = \frac{V^2}{\partial V/\partial n}, \qquad R = -\frac{V}{\partial V/\partial n}$$

where V is the potential at the point P. Now the element of distance ds of the space-time manifold depends on the potential V of the gravitational field. We should therefore expect that the element of distance ds *at the point* P would be that due to the mass C_ρ/K located at O, which is given by the Schwarzschild solution. This leads to the form

$$ds^2 = \left(1 + \frac{2V}{c^2}\right) dl^2 - (dx^2 + dy^2 + dz^2) - \left(\frac{1}{1+2V/c^2} - 1\right)(\alpha dx + \beta dy + \gamma dz)^2 \qquad 3.2$$

with reference to rectangular coördinates. (α, β, γ) are the direction cosines of the normal to the equipotential surface at the point (x, y, z). But the form (3.2) will not satisfy the field equations although it is exactly the form that we must have if a solution of the general case is to exist which is of the same *nature* as the Schwarzschild solution for a single particle. (3.2) appears to be more justifiable than the approximation given by Eddington (Cf. Eddington, *The Mathematical Theory of Relativity*, 1923, §46, equ. 46.2) since this form is not of the nature of an

approximation. According to the usual custom we would consider that the trajectories of the motion of a mass particle in the gravitational field are geodesics in the Riemann space defined by (3.2) i.e., that the Γ's in (1.1) are Christoffel symbols based on the form (3.2). In the following paragraph, however, we are giving a different method for the calculation of the Γ's.

4. Let us assume that all the quantities of the gravitational field, i.e., g_{ij}, Γ^i_{jk}, B^i_{jkl} B_{ij}, etc., at any point P are the same as though due to the mass C_p/K at the point O. The quantities of the gravitational field at the point P can then be calculated from the form

$$ds^2 = \left(1 - \frac{2C_p}{c^2 r}\right)dl^2 - (dx^2 + dy^2 + dz^2) - \frac{1}{r^2}\left(\frac{1}{1-2C_p/c^2 r} - 1\right) [(x-l)dx + (y-m)dy + (z-n)dz]^2 \quad 4.1$$

where $r^2 = (x-l)^2 + (y-m)^2 + (z-n)^2$; (l,m,n) are the coördinates of the point O. In the differentiations which are involved in calculating the quantities of the gravitational field C_p and (l,m,n) are to be treated as constants. If after all differentiations we replace C_p and (l,m,n) by their values as functions of (x,y,z) we shall obtain the quantities of the gravitational field as functions of the coördinates. *These quantities will satisfy the field equations* (1.2). The use of these values of the Γ's to describe the motion of a mass particle in the gravitational field according to the equations (1.1) would seem to express the principle that the force on the particle at the point P is due to the mass C_p/K at the point O associated with P. We cannot say, however, that the Γ's and B's are derivable from a fundamental tensor g_{ij}.

5. We have considered that the matter producing the gravitational field is stationary. If more than one body is present, however, the matter will move in the gravitational which it creates. The only example of such a system in nature is the universe itself. In this case we consider the motion of the matter as known and determine the Newtonian potential V which will now involve the time. The equipotential surfaces (3.1) will be equipotential surfaces at a particular time l. The quantities of the gravitational field will depend on the time but in the calculation of these quantities from the form (4.1) the time will be held constant during differentiation.

6. The assumption on which the calculation of the Γ's that we have just proposed depends, is essentially the following: *In an infinitely small space-time region of the gravitational field a coördinate system can be selected with reference to which space possesses a Schwarzschild structure.* The particular coördinate system associated with the neighborhood of any point P will be the spherical coördinate system (or its equivalent) with center on the normal to the equipotential surface through the point P

at the point O, which is a distance R from P. In case of a varying potential field our assumption requires the time to be held constant throughout the neighborhood of P, so that the quantities of the gravitational field are calculated as explained in the preceding paragraph.

In this connection we recall a very similar assumption which is made in the general theory of relativity, namely, that in infinitely small regions, and for coördinate systems with reference to which the bodies considered are unaccelerated, the special theory of relativity holds. This contains the assumption that space is euclidean with respect to this coördinate system. But we cannot use these coördinate systems here since their determination would require a knowledge of the motion of a particle in the gravitational field which we wish to calculate.

7. The equations (1.1) define a geometry of paths and give a definition of covariant differentiation based on the functions Γ^i_{jk}. (Cf. Eisenhart and Veblen, these PROCEEDINGS, **8**, pp. 19–23 (1922)). This geometry will become a Riemann geometry, i.e., the paths will become geodesics with respect to a tensor g_{ij} if

$$g_{ij,k} = 0$$

where $g_{ij,k}$ is the covariant derivative of g_{ij}, i.e.,

$$g_{ij,k} = \frac{\partial g_{ij}}{\partial x^k} - g_{\alpha j}\Gamma^\alpha_{ik} - g_{i\alpha}\Gamma^\alpha_{jk}.$$

The necessary and sufficient condition for the existence of the tensor g_{ij} (cf. Veblen and Thomas, "The Geometry of Paths," *Trans. Amer. Math. Soc.*, to appear in 1924) is the algebraic consistency of the infinite set of equations

$$\begin{aligned} g_{i\alpha}B^\alpha_{jkl} + g_{\alpha j}B^\alpha_{ikl} &= 0 \\ g_{i\alpha}B^\alpha_{jklm_1} + g_{\alpha j}B^\alpha_{iklm_1} &= 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \qquad (7.1) \\ g_{i\alpha}B^\alpha_{jklm_1\cdots m_n} + g_{\alpha j}B^\alpha_{iklm_1\cdots m_n} &= 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \end{aligned}$$

where $B^\alpha_{jklm_1\cdots m_n}$ is the nth covariant derivative of B^α_{jkl}. Hence, *the space-time trajectories of the motion of a mass particle in a gravitational field which depend on* Γ*'s calculated from* (4.1) *will be geodesics with respect to some tensor* g_{ij} *if and only if the sequence* (7.1) *be algebraically consistent.*

I wish to express my thanks to Professors Eisenhart and Hille who have read this paper and who have improved it by their suggestions.

* NATIONAL RESEARCH FELLOW IN MATHEMATICAL PHYSICS.

THE ROOT SYSTEM OF EPIGAEA REPENS AND ITS RELATION TO THE FUNGI OF THE HUMUS

BY W. T. COUNCILMAN

HARVARD MEDICAL SCHOOL AND ARNOLD ARBORETUM, BOSTON

Communicated July 3, 1923

The Epigaea repens, an evergreen woody shrub, known commonly as the Mayflower and trailing arbutus,* has a small hard irregular, often twisted and knobby, underground stem from 2–4 cm. in length and rarely more than $1^1/_2$ cm. in diameter. From this stem are given off numbers of coarse hard roots, dark brown in color, which extend both laterally and in depth, and from these smaller roots arise. The larger roots do not terminate in a mass of capillary roots; they end abruptly and are often broken and dead at the ends. From all of these roots and from the stem both terminals or capillary roots in great number arise; either short branches which quickly break up into a series of terminals are given off or a branch may run a considerable distance before terminating in capillaries. The roots forming the capillaries even when given off low down tend to approach and ramify in the humus and the greatest number are found in or just below the surface. They are added to by bunches of roots given off from the branches which for a considerable distance from the central stem of origin are in contact with the surface. The result is that the humus and the upper soil for a diameter nearly that of the entire plant are filled with great numbers of fine roots. They are very fragile and, being closely intermingled with the humus particles and with the general root labyrinth of the humus, it is difficult to isolate them. The most careful teasing and washing will give but a part of the terminals of a single plant. The single roots of the terminals often extend for considerable distances (3–4 cm.) without branching giving to the mass when floated in water a length of even 8–10 cm. Fig. 1. The final terminals are extraordinarily small, often too small to be seen singly by the naked eye and only 30–40μ in diameter as actually measured. No root cap is found and although the pointed tip with closely appressed following cells may be found, especially in the short laterals, the end is most frequently irregular, appearing broken.

The Epigaea is essentially a plant of the forest humus but it does not flourish in deep shade. It is more commonly found at the edge of the forest or alongside of a wood road and also where a pine forest has been cut down and the second growth has not become sufficiently dense to give the unfavorable condition of complete shade. I have found it in great abundance in a comparatively thin hardwood forest with high shade. It is frequently found in company with the Gaultheria procumbens which seeks the same conditions and as the roots have much similarity, even in

the matter of fungus relations, they may be mistaken for one another. The Gaultheria is much more common being protected by the absence of the fatal gift of conspicuous beauty.† The Epigaea was formerly found in great abundance on Cape Cod growing at the edge of and in the thin forest of pines and oaks. In many places on the Cape it has extended for some distance into the adjacent fields which are covered with a thin bunchy grass, but these outlying plants I have always found to be inferior specimens. The character of the soil does not seem to make any material

FIGURE 1

difference; it grows equally well in stiff clay, in sandy or micaceous loam or in the Cape Cod sand, the single condition being the overlying humus. The apparent exception, that of the growth in the open on Cape Cod, is not really an exception for it does not stray beyond the thin humus formed by grasses and blown leaves which is adjacent to the forest. The illustration of roots is from a vigorous plant from the edge of pine forest on Cape Cod.

The roots are entirely devoid of even the most rudimentary root hairs and no plant is more favorable for the study of the relation between the roots and fungus. There is considerable choice in the period of the year.

The fungus though always present is more abundant in the late summer and early autumn. A mass of the fine roots examined under a 16 or 8 mm. lens forms a striking microscopic object. The roots, particularly the fine terminals, are clear in color, the light brown shade being due to the accompanying colored hyphae of the fungus. They are simple in structure, composed of square or oblong epidermis cells enclosing the vascular tissue. The finest roots have no more than four of these surrounding cells, the larger have more but the cells do not vary in size and structure. They have thin walls as compared with roots of other Ericaceae, such as Calluna and Gaultheria, and it is difficult to dehydrate the cells for staining and sectioning.

On these roots a great number of brown hyphae of a fungus are found forming a plexus with numerous communications, although single hyphae often run long distances without branching. Fig. 2. This mycelium is

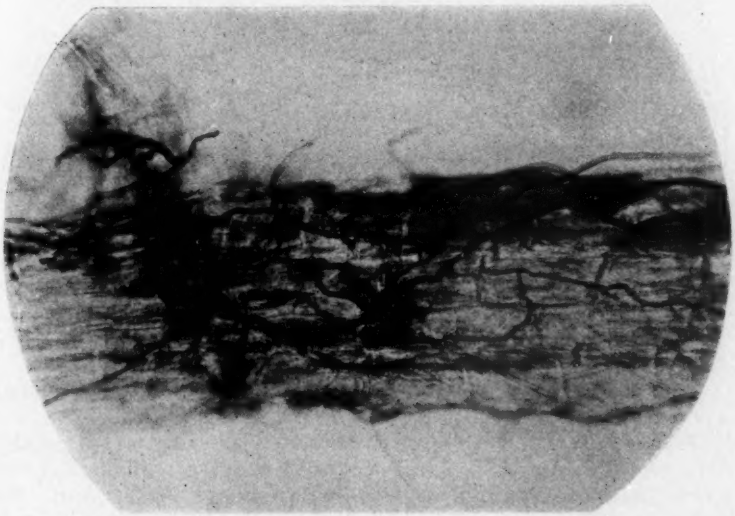


FIGURE 2

abundant on the larger roots but reaches its greatest development on the fine capillary roots where it may form so close a surrounding mass that the single hyphae are distinguished with difficulty. Fig. 3. In all places, but particularly where most abundant, the hyphae penetrate between the covering cells and may extend in the tissue beneath them. From the surface and from within the tissue of the root, the hyphae give off fine haustoria, often with clubbed ends which penetrate the cells. The cell contents in contact with the fungus appears more granular and the cells

larger. In many cells the penetrating hyphae divide rapidly and form a mass filling the cell and closely resembling the mass of fine capillary blood vessels composing the glomerulus of the kidney. Fig. 4. The term glomerulus can fitly be given to the structure. The invaded cells vary greatly in numbers, in places they are here and there only, but in others almost every cell is affected as towards the end of the root or where there are numerous branches. The glomerular structure is formed not so much by the coiling of the hyphae within the cells as by rapid division of the small branches. All the hyphae on the outside of the root are dark brown or even black but those forming the intracellular mass are devoid of color although the entering haustoria retains their color. The beginning

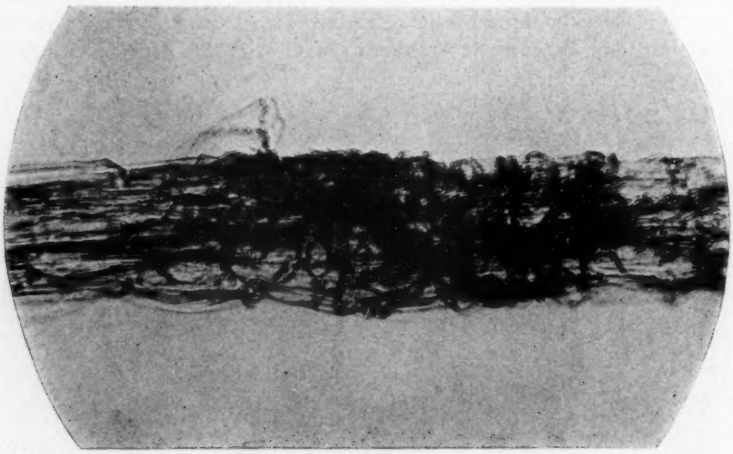


FIGURE 3

of glomerular formation is often seen, the entering hyphae become swollen and give off a number of small branches which divide. The inner cell wall may become covered with hyphae before the central mass is formed. The hyphae which penetrate between the cells and beneath them are paler than those on the exterior of the root. There is but little difference in the size of the hyphae, the usual diameter being 0.8μ but occasionally twice this size may be found.

In the humus about the roots as in forest humus generally there is a great deal of black or brown or colorless mycelium in the form of loose masses or the very peculiar ropes of twisted hyphae or rhizomorphs. It has never seemed to me that there is a close relation between the fungus of the root and the masses in the adjacent humus. Hyphae do not pass from the soil into the root nor are they given off from the root to mingle

with the mass in the humus. There is no difficulty in separating the roots from the fungus masses in the soil as is often the case in the mycorhizas of forest trees. Bits of humus are often entangled in the roots but they are not bound to them by mycelium. Occasionally one encounters areas on the root where the closely investing mycelium partakes of the form of a mycorhiza, forming a close network with numerous communications, but outstreaming hyphae are never given off as in the true mycorhiza. I have, however, found in other Ericaceae, as Pieris floribunda, short pro-

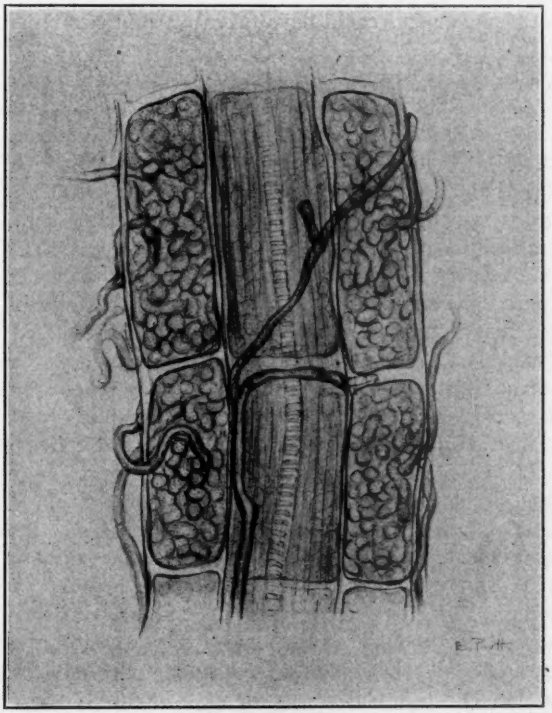


FIGURE 4

jecting hyphae with a spreading end where attached, very similar to the mycorhiza. I have never been able to assure myself of the presence of fruiting forms of the fungus in the root. In one plant I have found within the cells small globular bodies closely resembling those found in the cells of the root tubercles of Alnus, but could not determine their nature.

Plants examined from different localities in Maine, Massachusetts and Maryland show always apparently the same fungus in the same relation to

the roots. I have lately through the kindness of Professor Sargent of the Arnold Arboretum had the opportunity of examining a plant of Epigaea from northern Japan, which was preserved in the herbarium. A few fine roots still clung to one of the branches and on soaking these out in water there were found the same dark hyphae in the identical relation to the root, including even the intracellular growth, as in the American specimens. This goes to show that the relation of fungus to plant prevailed in the plants at their common source before the last glacial period, for it could hardly be supposed that a process of evolution involving the relationship of both plant and fungus could have taken place in two remote and unrelated regions.

It cannot be assumed that the relationship between plant and fungus is pathogenic. No plant is without the fungus and the more vigorous the plant the more abundant the fungus; the necessity for humus may be that this gives suitable conditions for the growth of the fungus. There is great need in the study of the nutrition of plants, of pure cultures of the fungi of the humus, which seems at present an unattainable end. One has, however, in studying the relation the strong impression that the fungus is as definite an entity as the plant and the union not a chance marriage. Without pure cultures of the fungi and the experimental study of the relationship it is impossible to decide this and many related questions. In a note made last October, I find the following question. "Is it possible that the intracellular bodies represent a food supply for the plant which it used up in such special activities as blooming and seeding?" Plants taken from the same locality just before and during the blooming show a great difference in the fungus as compared with the summer and autumn. But few well developed glomerular masses are found within the cells but in place of these are cells containing material which has resulted from the softening and disintegration of the hyphal mass. There is in some of these cells merely a small amount of indefinite granular material and colored granules, in others along with the granular material there are fragments of swollen hyphae. Into some of these disintegrating cells new hyphae were entering and dividing to form new glomerular masses.

The relation between plant and fungus is one of perfect symbiosis, they live together mutually benefitting but it would be difficult to say just what each gives the other. Possibly one of the functions of the fungus is to tide over by the use of material resulting from disintegration, a period of increased nutritive demand and it may also serve through its large surface the place of the root hairs in taking from the soil water and salts. Frank, in his first admirable description of the root fungi, spoke of them as playing the part of a wet nurse for the plant. The plant on its part may give not only the suitable physical conditions for growth of the fungus but may contribute to this by its nutritive activities. There is too little

known in regard to the soil fungi to make speculation as to their action profitable.

* Both names are singularly ill chosen for, not being indigenous to England, it is not the Mayflower there, and it is uncertain whether this or some other of the early spring flowers was the Mayflower of the colonists. It is found only in America where it widely distributed, and in Japan. It has no relation with Arbutus save in that both belong to the Ericaceae.

† The Epigaea repens is one of the most beautiful and interesting of plants. Its blossoms which are among the earliest of the spring flowers are white or pink with a waxy texture and a delicious spicy odor and are borne at the extremities of the stems. The pale green hairy leaves and the pale pink or green stems streaming from the centre close upon the surface of the soil add to the attractiveness. The environment of dead brown leaves, mosses and low plants gives a perfect setting. It is unfortunate that these wonderful qualities should be those which are ensuring the destruction of the plant. Large quantities are gathered in the spring and hawked around the city streets, the unfortunate city dwellers seeking to satisfy atavistic and misunderstood yearnings for woods and green dales by purchasing the bunches which to one knowing the plant in its habitat seem very unattractive. The exploitation is ruthless; the branches bearing many of the fine roots are torn from the stem. The traffic is of great extent; where the plants are abundant a family even selling the bunches at wholesale can often earn $25.00 a day but with the declining plants the remuneration is not so great. The plant is of slow growth and the relation of leaf and root is so finely adjusted that recovery after considerable loss does not take place and the stimulation to effect new growth cannot act on the plant and the fungus at the same time. By great care and skill plants can be transferred to other suitable localities and may even be propagated by seed but there is little prospect of its ever becoming a garden habitant. Although under natural condition it spreads laterally by means of the branches giving off roots and forming new centres, and by means of seed which are abundantly formed on the female plants, growth is slow, the seeds do not easily germinate. It is also destroyed and made impossible by the forest fires which consume the essential humus. The automobile by rendering remote places easily accessible, has contributed greatly to its destruction, but the most powerful agency is the commercial exploitation. I have known it to completely disappear from localities where formerly common and probably no plants can now be found within a dozen miles of any of the large cities. This desire to save the plant is not a mere matter of sentiment, no plant is more available for study of the relation between plant and fungus, and for awakening in children through its study the all important wonder and curiosity. It is safe to say that within 20 years the plant will have disappeared from most of the places where now found, although it may be preserved on large estates from which the public are excluded. Apparently like all wild beautiful things which man covets it must go but the loss of such things is a serious loss for man.

THE SYMMETRY OF THE INTERNAL EARS IN FLATFISHES

BY J. V. LEECH

ZOÖLOGICAL LABORATORY, HARVARD UNIVERSITY

Communicated June 26, 1923

Flatfishes, such as the flounder, have attracted a great amount of attention because of their economic and scientific importance. They have been of particular interest to zoölogists because of certain morphological changes which take place during growth and development. When the young flatfish hatches, its eyes are on the right and left sides of its head as in ordinary fishes, but as development proceeds, one eye migrates to the opposite side so that in the adult both eyes are on the same side of the head. Traquair (1865) gave the first accurate account of this asymmetry. By an exhaustive examination of the skull and lateral line system, he was able to trace the exact course followed by the head in its development to the adult form.

The present study was made to determine whether or not an asymmetrical condition existed in the gross morphology of the ears of flatfishes. The problem suggests itself by virtue of the relative positions of the ears in the adult fish. Curiously enough, larval flatfishes maintain their bilateral symmetry and upright position for a number of days after hatching. Cole and Johnstone (1902) have described at length the life history and habits of Pleuronectes platessa. According to them this species maintains its bilaterality and upright position up to thirty days after hatching. At this age it is quite similar to ordinary symmetrical teleostian larvae. Gradually the left eye begins to move around the head and by the forty-fifth day it has attained its permanent position on the right side of the head. During the time in which the eye is migrating, the young fish gradually acquires a new position in swimming so that finally it swims with its lateral eyeless side down. Thus gradually the whole body of the fish is rotated through an angle of 90 degrees and the animal now rests and swims on its left side.

The internal ears, imbedded in the head of the fish, rotate with the body so that the left ear remains on the eyeless or left side while the right ear is above it on the ocular side. Hence all parts of the ear are at right angles to their former position in relation to the pull of gravity. The ears of fishes are well known to be organs of equilibration and to have to do with maintaining the animal in an upright position. Maxwell (1922) has recently given an interesting account of this phase of the subject. In the flatfishes the rotation of the body must involve a rotation of all those organs of the ear that are concerned with equilibrium. Do these organs show any adaptation to this change of position?

The two species used in this study were Pseudopleuronectes americanus and Limanda ferruginea, common forms from the New England coast. In these fishes, as in other teleosts, the ear is covered externally by bone. Internally, it projects freely into the cavity of the cranium, being separated from the brain by membranes only. It is possible by careful dissection to remove the ear and the eighth nerve of such a fish in toto. Numerous dissections of this kind were made and the preparations were subsequently stained in $^1/_{20}$ per cent osmic acid. Thus the ears were prepared for a close comparative study.

The following description applies particularly to Limanda ferruginea. The internal ear of this fish consists of the utriculus with its sinus superior and recessus utriculi, three semicircular canals with their corresponding ampullae, the sacculus and the lagena. Three otoliths are present, one in the recessus utriculi, another in the lagena, and a third very large one in the sacculus. There are six end organs namely, the macula acustica recessus utriculi, three cristae acusticae ampullarum, the macula acustica sacculi and the papilla acustica lagenae. The macula neglecta is absent. The eighth nerve (acusticus) is very short and gives off a ramus anterior which branches into the ramulus ampulla anterioris, the ramulus ampulla externa and the ramulus recessus utriculi. The ramus posterior branches into the ramulus sacculi, the ramulus lagenae and the ramulus ampullae posterioris. The utriculus is a narrow sac lying horizontally. It is bent outward at each end and is expanded anteriorly into the recessus utriculi, on the base of which is carried the macula recessus utriculi. A small irregularly shaped otolith lies freely in the cavity of the recessus. The ampulla externa arises laterally from this expanded region. It is of usual shape and bears the usual crista acustica. Arising dorsally from the anterior end is the ampulla anterioris. It is likewise of characteristic shape but it is placed vertically and bears its own end organ. A slight enlargement of the posterior end of the utriculus gives rise to a posterior sinus. It receives the horizontal semicircular canal and dorsal to its opening gives rise to the posterior ampulla. Its shape is similar to that of the other two ampullae and, like the anterior one, it is placed vertically. The sinus superior arises from the median dorsal surface of the utriculus. Its general shape is that of a somewhat flattened cone, the apex of which is formed by the union of the anterior and posterior semicircular canals.

The sacculus lies closely applied to the under surface of the utriculus. It is oval in shape and is compressed laterally. The macula acustica sacculi, the largest end organ of the ear, is located on its medial surface. A large otolith conforming to the general shape of the sacculus occupies the larger portion of the saccular cavity. The utriculo-saccular opening permits free communication between utriculus and sacculus. The lagena is a pocket-like outgrowth from the posterior dorsal angle of the sacculus.

There is free communication between it and the sacculus. In its cavity is a small irregular otolith. The papilla acustica lagenae occupies a position on the medial surface of the lagena.

The right and left ears of Limanda and Pseudopleuronectes were closely compared in the various details of their anatomy as described in the previous paragraph. In the branching of the right and left eighth nerves, the form of the two ear sacs, the shape, size, and position of their end-organs, the right and left ears of these fishes were found to be exactly symmetrical, a condition that confirms the general statement made by Retzius (1881) that the internal ears in the flatfishes are alike.

When it is recalled that with the rotation of the body of the flatfish the corresponding otoliths in the two ears must come to rest in unsymmetrical positions in relation to the ear sacs themselves, it is obvious that the ears of these fishes must function in a very remarkable way. Thus in any flatfish the saccular otolith of the lower ear must rest under the influence of gravity on the outer or non-nervous side of its containing organ, while that of the upper ear must rest on the nervous side of the sacculus of that side. Such conditions are very difficult to interpret from a physiological standpoint and invite investigation. They are suggestive of Maxwell's general view that the stimulus to equilibrium is a deformation of the ear sac as a whole rather than a specific action of particular parts. But, as already intimated, this is a problem for further inquiry.

Cole, F. J., and J. Johnstone, Pleuronectes (the plaice). *Proc. Trans. Liverpool Biol. Soc.*, **16**, 1902 (145–396).

Maxwell, S. S., *Labyrinth and Equilibrium*, 1922.

Retzius, G., *Das Gehörorgan der Wirbelthiere. I. Das Gehörorgan der Fische und Amphibian*, 1881.

Traquair, R. H., On the Asymmetry of the Pleuronectidae as Elucidated by an Examination of the Skeleton in the Turbot, Halibut and Plaice. *Trans. Linnean Soc., London*, **25**, 1865 (263–296).

MOTION OF AN ELECTRIC PARTICLE IN A RIEMANN SPACE

BY ARTHUR BRAMLEY

DEPARTMENT OF PHYSICS, PRINCETON UNIVERSITY

Communicated June 17, 1923

Consider a Riemann space with linear element

$$ds^2 = g_{ij}\, dx^i\, dx^j$$

then the equations of motion of a charged particle can be written in the form (Eisenhart, THESE PROCEEDINGS, **9**, pp. 175–8 (1913)

$$g_{ij}\left(\frac{d^2x^j}{ds^2} + \Gamma^j_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}\right) = \phi_i,$$

where

$$\Gamma^j_{\alpha\beta} = \frac{1}{2}\, g^{kj}\left(\frac{\partial g_{\alpha k}}{\partial x^\beta} + \frac{\partial g_{\beta k}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^k}\right),$$

and the vector ϕ_i is defined as the ponderomotive force of the electromagnetic field

$$\phi_i = F_{ik}J^k$$

where F_{ij} are the components of the electric and magnetic intensities and

$$J^k = \rho_0\left(\frac{dx^1}{ds}, \frac{dx^2}{ds}, \frac{dx^3}{ds}, \frac{dx^4}{ds}\right)$$

is the current vector.

This definition of ϕ_i enables us to determine the ϕ's for any given distribution of charge density. Consider the case of a single positive charge, e.g., atomic nucleus.

The fundamental form is evidently,

$$-ds^2 = \left(1 - \frac{2e}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 - \left(1 - \frac{2e}{r}\right) dt^2$$

where e = charge constant.

Since we can take the charge as fixed at the origin, we have

$$\frac{dx^1}{ds} = \frac{dx^2}{ds} = \frac{dx^3}{ds} = 0 \quad \text{and} \quad \frac{dt}{ds} = 1;$$

then

$$g_{ij}\,\Gamma^j_{44} = \phi_i = F_{ik}J^k.$$

The only equation not zero, is

$$g_{11}\Gamma^1_{44} = \phi_1 = F_{14}J^4 = F_{14}\rho_0$$

for

$$J^k = \rho_0(0, 0, 0, 1), \qquad \phi_1 = g_{11}\,\Gamma^1_{44} = -\frac{2e}{r^2}$$

$$\therefore F_{14} = -F_{41} = -\left(\frac{2e}{\rho_0}\right)\frac{1}{r^2} = \frac{-\epsilon}{r^2}.$$

The law of force derived in this case is the well known Coulomb's law, according to which the force varies inversely as the square of the distance; the charge on the attracting particle is

$$\epsilon = 2e/\rho_0$$

Having determined the ϕ's and g's for a positive charge, the path of any charged particle under the action of the positive charge can be easily found. Let us consider the case of an infinitesimal charge, we have

$$\phi_i = F_{ik}J^k, \qquad \phi_1 = F_{14}\bar{\rho}_0\frac{dt}{ds}, \qquad \phi_4 = F_{41}\bar{\rho}_0\frac{dr}{ds}$$

the other equations being identically satisfied.

Since

$$\phi_2 = \phi_3 = \phi_4 = 0$$

and

$$\phi_1 = \bar{\rho}_0 F_{14}\frac{dt}{ds} = \rho_0 F_{14} = g_{11}\,\Gamma^1_{44},$$

we have

$$\frac{dr}{ds} = 0 \text{ or } \frac{dr}{dt} = 0,$$

where $\bar{\rho}_0$ = the charge on the central body.

These equations have a direct bearing on the quantum theory.

From $dr/dt = 0$, we see that the paths of a particle under the action of the inverse square law can be only circles, i.e., there can be no radiation of energy. For (Eddington, *Report*, p. 49) the equations of motion of the particle are (μ = mass of particle)

$$(1) \qquad \mu g_{11}\left[\frac{dr^2}{ds^2} + \frac{1}{2}\lambda'\left(\frac{dr}{ds}\right)^2 - re^{-\lambda}\left(\frac{d\phi}{ds}\right)^2 + \frac{1}{2}e^{\nu-\lambda}\,\nu'\left(\frac{dt}{ds}\right)^2\right] = \phi_1$$

$$(2) \qquad \frac{d^2\phi}{ds^2} + \frac{2}{r}\frac{dr}{ds}\frac{d\phi}{ds} = 0, \qquad (3) \quad \frac{d^2t}{d^2s} + \nu'\frac{dr}{ds}\frac{dt}{ds} = 0$$

$$(4) \qquad \frac{d^2\theta}{ds^2} - \cos\theta\sin\theta\left(\frac{d\phi}{ds}\right)^2 + \frac{2}{r}\frac{dr}{ds}\frac{d\theta}{ds} = 0$$

Since $dr/ds = 0$, we have from (2)

$d\phi/ds = \text{const.} = 0$ (particular intial value)

From (3) we see that

$$dt/ds = \text{const.} = k$$

From (1)

$$\tfrac{1}{2} e^{\nu-\lambda} \nu' k^2 = \frac{1}{2} e^{\nu-\lambda} \nu' \frac{1}{\mu},$$

which gives

$$k^2 = 1/\mu$$

From (4)

$$\frac{d\theta}{ds} = k_1 \text{ or } \theta = \mu^{-\frac{1}{2}} At + B \qquad A \text{ and } B \text{ constants.}$$

whence the particle moves with uniform velocity in a circular orbit about the positive charge.

The velocity is evidently

$$r\frac{d\theta}{ds} = v\sqrt{1-\gamma}\cdot\frac{1}{\mu}\sqrt{-1} \qquad \gamma = \left(1-\frac{2e}{r}\right)_{r=\text{const.}}$$

the unit being the velocity of light. Thus the frequency depends on the ratio of the masses of the charges.

The fact that an infinitesimal particle revolving about the atomic nucleus, must describe a definite orbit with constant velocity is rather remarkable. Although the above considerations only apply to an infinitesimal charge, a similar occurrence will probably happen in the case of an electron revolving about the nucleus. These considerations are in agreement with the quantum theory expounded by Bohr. In fact, he postulated as the fundamental hypothesis for interatomic mechanics the type of orbit we have just considered.

In the case of radiating charges, we cannot say that the path is a geodesic in the space determined by the μ and the ϵ of the central particle, so that the above method is not applicable to radiating electrons; but some other type of laws must be used other than the classical equations of electrodynamics.

THE MARGINAL BELTS OF THE CORAL SEAS

By W. M. Davis

Harvard University

Read by title before the Academy, April 23, 1923

1. The marginal belts of the coral seas, having a probable width of about 5° between latitudes 25° and 30° north and south of the equator in the central and western parts of the Pacific ocean, and apparently looping across the equator near the Marquesas islands in the eastern half of this ocean, are characterized by a small number of strongly clift volcanic islands or islets surrounded by submarine banks of small depth and from 10 to 60 miles in diameter, some of which are surmounted by coral reefs.

The volcanic islands of the cooler seas resemble the marginal-belt islands in being clift, but differ from them in not being associated with extensive banks or with coral reefs. The volcanic islands of the warmer or coral seas resemble the marginal-belt islands in being as a rule associated with bank-like lagoon floors rimmed by coral reefs, but differ from them in having with few exceptions, non-clift shores.

2. The Pacific ocean therefore possesses three categories of islands, besides the formerly glaciated islands of the colder seas which are not here considered; and these three island categories appear to be related to three climatically defined zones: the warmer or coral seas, which have been persistently warm enough for reef growth in later geological time, even during the Glacial epochs of the Glacial period; the cooler seas, which include the eastern equatorial Pacific, and which have been as persistently too cool for coral growth; and an intermediate area, here called the marginal belts of the coral seas, in which reefs have grown up to normal ocean level in epochs of warmer or non-Glacial climate, but in which reef growth has been inhibited and the reefs have been cut away and the islands behind them have been attacked by the waves of the lowered and chilled ocean in epochs of Glacial climate.

3. Theories of coral-reef formation may be divided into two main groups: First, those which postulate prevailingly stable reef foundations; the Glacial-control theory as elaborated by Daly is much the best argued theory of this group. Second, a single theory, Darwin's, which postulates unstable reef foundations and associates the formation of barrier reefs and atolls with slowly or intermittently subsiding foundations, usually volcanic islands.

Besides postulating stable reef formations, the Glacial control theory introduces new and important elements into the old coral-reef problem by considering changes of ocean level and of ocean temperature, as well

as the abrasional consequences thereof, as intimated at the end of the preceding section. But in combining these new elements only with the postulate of stable islands their possible value is limited; they will be here combined with the postulate of unstable islands also. The object of this essay is therefore to determine, if possible, whether the features of the three categories of Pacific islands are better explained by the postulate of insular stability or of insular instability, each postulate being considered in association with the Glacial-control changes of ocean level and temperature and their consequences.

4. If the volcanic islands of the cooler Pacific seas are as a rule stable, the older ones ought to be completely or almost completely truncated by normal supplemented by low-level abrasion, so that the resulting normal or low-level rock platforms, extended seaward by detrital embankments and now more or less aggraded with Post-glacial organic and inorganic deposits, should appear as banks comparable in area and depth with those of the completely or almost completely truncated islands above noted as occurring in the marginal belts. But no such banks are found in the cooler seas of the Pacific; hence it must be supposed either that those seas do not contain any islands old enough to have been thus truncated and this seems improbable; or that such old islands as they do contain have not been stable.

5. If old unstable islands in the cooler seas have long been slowly or intermittently rising, they would now appear as terraced cones, the upper terraces being more or less delapidated by erosion; but no such islands are known in the cooler Pacific seas. If the islands have long been sinking, they would eventually take the form of submerged terraced cones, on which the earlier and lower terraces would be obscured by detritus from the later and higher terraces. If a sinking island were almost truncated by an uppermost terrace, its surviving remnant would rise as a stack or pinnacle from near the terrace center; and the terrace might then be sunk to a considerable depth while the ocean waves were beating rather ineffectually on the plunging pinnacle cliffs. But pinnacle islands of this kind are rare in the cooler seas, and the few that are known are surrounded by relatively deep water.

If a subsiding island were completely truncated by the uppermost terrace, that terrace might now be submerged to any depth and would be known only by soundings. So few soundings have been made in the cooler seas of the Pacific that submerged truncated islands still may exist there in considerable numbers as yet undiscovered. The rarity of pinnacle islands and shallow banks in these seas as compared with the abundance of atolls in the warmer seas, may therefore be plausibly ascribed to the destruction of the cooler-sea islands by abrasion and their concealment by subsidence. But their rarity may possibly be the result of the rare

production of volcanic islands in the cooler seas, for such islands are not necessarily distributed evenly over the ocean floor. Exploration of the ocean by echo-soundings may resolve this uncertainty.

6. The absence of extensive shallow banks around the few clift islands of the cooler Pacific seas having led to the inference that the islands of those seas are unstable with a tendency to subsidence, the islands of the marginal belts may be provisionally regarded as of similar behavior. Thus regarded, the extensive banks around them should not be interpreted as shallow abrasional platforms of volcanic rock thinly veneered with organic and inorganic deposits, but as largely composed of coral-reef rock and lagoon-floor beds, built up on subsiding volcanic foundations with respect to normal ocean level in former non-Glacial epochs, cut away by low-level abrasion in Glacial epochs, and somewhat aggraded and imperfectly built up again by reefs in the present Post-glacial epoch. As such, these banks would realize the expectations of the Glacial-control theory, except that the reef foundation would not be stable, and that the oceanic area in which such banks occur is rather narrowly restricted instead of being widely expanded.

7. The most remarkable series of clift marginal-belt islands and shallow banks in the Pacific is found in the northwestern extension of the Hawaiian chain. The banks are from 10 to 30 miles in diameter, and are seldom over 30 or 40 fathoms in depth. When reefs occur, they stand back from the margin of the banks and may therefore be called bank-atolls, or bank-barriers in case a central island survives. The Hawaiian series includes 6 bank-atolls, 3 banks without reefs but with clift central islands or pinnacles, 5 banks without reefs or central islands, and 2 intermediate forms.

In the South Pacific, Norfolk island, north of New Zealand in latitude 29° S., is about 13 square miles in area and 1050 feet in height; it is strongly clift and is partly bordered by a small coral reef. The smaller Phillip island, also strongly clift, is 3 miles away. Both islands stand near the center of a vast bank, 60 by 20 miles in extent, with depths of 20 fathoms near the islands and 40 or 50 fathoms near the outer margin. The absence of embayments in this fair-sized island suggests relatively slow subsidence and strong abrasion. Lord Howe island, between Norfolk island and Australia in 32° S., is of smaller size with cliffs 800 feet high; it rises from a bank 12 miles by 8 miles in extent and similar to the Norfolk bank in depth. Balls pyramid, not far away, is an extraordinary stack or pinnacle, its height of 1816 feet being greater than its shorter sea-level diameter; it stands on a bank 3 by 7 miles in extent. Middleton and Elizabeth reefs are small bank-atolls, north of Lord Howe island in latitude 29° S.

The absence of similarly clift islands rising from extensive banks in the cooler seas, and the occurrence of the above named islands and banks along the margin of the coral seas, as well as the Marquesas mentioned

below and a few others elsewhere specified,[1] are believed to constitute highly significant elements of the coral-reef problem.

8. The inference above reached regarding the instability of Pacific islands in the cooler seas and the marginal belts of the coral seas is strongly confirmed when the reef-encircled islands of the Pacific coral seas are examined. In the first place, these reef-encircled islands are, with few exceptions, not clift; their sculpture appears to be wholly the work of subaerial erosion. Even in the case of islands bordered only by fringing reefs which fall off into deep water, the island spurs slope gradually to the shore. Hence although the lowering of the Glacial ocean in the Glacial epochs must have left these and many other islands for a time somewhat more emerged than now, it must be supposed that their exterior reef slopes were continually protected by living corals at and a little below the lowered ocean level, and that low-level abrasion was prevented then just as normal abrasion is prevented now, as has been implied above.

In the second place the reef-encircled islands of the coral seas are as a rule well embayed; and the form of the embayments cannot be adequately explained by the relatively short-lived erosion of stable islands with respect to the lowered ocean of the Glacial epochs, particularly in the case of mountainous islands having barrier reefs a mile or more from their shores: for the inferred rock-bottom depth—not the water depth—at the mouth of the embayment in such islands is decidedly greater than can be accounted for by erosion by steep-falling mountain streams, heavily charged with coarse detritus and mouthing in the lowered ocean on the outer face of the barrier reef a mile or more seaward from their former mouths.

9. Reef-encircled clift islands of the coral seas are few in number.[2] They all seem to be young islands. They must have been free from protecting reefs while the cliffing was in progress; and their freedom from reefs at that time may be ascribed to the abundant detritus swept down by their steep young streams during a time of temporary stability, whereby a beach of moving cobbles and gravels was formed around their shores on which corals could not establish themselves. Then if subsidence sets in, the valleys will be somewhat embayed, the stream-washed detritus will be pocketed in the embayments, and reef growth may begin, and may be continued during later subsidence. When the previously-cut cliffs are wholly submerged, the island spurs will slope gradually down into the widened lagoon waters, except for little nips cut at their ends by the lagoon waves.

10. The atolls and almost-atolls of the coral seas are best explained by upgrowth on subsiding foundations. The contrary explanation, that they surmount consumed or almost consumed stable islands is unsatisfactory, because no process for the complete consumption of stable

islands in the coral seas can account satisfactorily for the absence of large and shallow banks as the residual forms of similar islands in the cooler seas, or for the non-clift, mountain-top forms of the islets in the almost-atolls of the coral seas.[3] For if the atolls of the coral seas are built up from rock platforms cut across old, worn-down, deeply weathered, stable volcanic islands by low-level abrasion during the Glacial epochs, then similar islands in the cooler seas ought now to be represented by submarine banks of dimensions similar to atoll-lagoon floors; but, as above said, such banks are unknown. And, under the same assumption, certain other stable islands in the coral seas of somewhat greater size, or of less age, or of greater resistance than those now wholly truncated and surmounted by atolls, ought not yet to have been so completely truncated as the atoll platforms; and the residual islets of such longer-lived islands rising in almost-atoll lagoons, ought to be clift, like the residual islets of the marginal belts; but the islets of almost-atolls are not clift.

11. The foregoing review of the islands in the cooler seas, the marginal belts and the coral seas of the Pacific, here presented in condensed form after a careful study of all available data, supports the postulate of unstable islands in association with changes of ocean level and temperature; that is, Darwin's theory modified by Glacial-control factors: but the effects of these modifying factors are clearly recognized only in the marginal belts, while the consequences of Darwin's theory appear to be of equally essential value there and of dominant value in the much larger area of the Pacific coral seas.

[1] Drowned Coral Reefs South of Japan, these PROCEEDINGS, **9**, 1923 (58–62).

[2] Clift Islands in the Coral Seas, *Ibid.*, **2**, 1916 (283–288).

[3] The Small Islands of Almost-Atolls, *Nature*, **105**, 1920 (292–293).

THE DEPTH OF CORAL-REEF LAGOONS

BY W. M. DAVIS

HARVARD UNIVERSITY

Communicated June 17, 1923

1. The moderate depth of coral-reef lagoons and the roughly accordant depths of lagoons of similar area, as well as the smoothness of their floors, have been taken to prove that the lagoon floors of the coral seas cannot have been given their present form by the accumulation of detritus on subsiding foundations of uneven shape, as Darwin thought, but that they must be underlaid by nearly level rock platforms that were abraded

by the lowered and chilled ocean of the Glacial period across stable islands at a standard depth below normal ocean level.

The chief reasons, some old and some new, for rejecting this view and for holding, with subordinate modifications, to Darwin's theory of reef upgrowth on subsiding formations are here summarized, in order to complete the statement for that theory as presented in the foregoing article.

2. A weighty reason for rejecting the assumption that rock platforms abraded across stable islands serve as the foundation of atolls is, as already briefly stated, the absence of plunging cliffs on reef-fringed islands and on the islets of almost-atolls; for the abrasion that would completely truncate islands of weak rocks, 10 to 20 miles in diameter, such as are assumed under the Glacial-control theory to have been the Preglacial precursors of present-day atolls, would surely have cut strong cliffs on the spur ends or slopes of islands made of harder rocks. The well developed cliffs on the islands of the marginal belts, where low-level abrasion has manifestly been operative, confirm this view. Furthermore, if an embayed island within a barrier reef be assumed to be stable, the valleys now occupied by its embayments must have been eroded with reference to the lowered ocean of the Glacial epochs; but in addition to its being impossible that many actual embayments should have been eroded down to their inferred rock-bottom depth under such conditions, as already noted, it is also impossible that the now-embayed valleys should have gained their observed width by low-level erosion without having strong cliffs cut all around the island shores by contemporaneous low-level abrasion, if such abrasion were really in operation. Even if subsidence were then in progress, strong cliffs must have been produced if abrasion had been at work here as effectively as it was in the marginal belts. The absence of such cliffs proves that abrasion was not operative on the islands within the barrier reefs of the coral seas, and this shows that living reefs must have been present there; hence abrasion could not have been operative on the islands of those seas that are now represented by atolls and almost-atolls.

3. An additional reason for not believing in the existence of abraded rock platforms beneath the lagoon floors of sea-level atolls is that no trace of such rock platforms is found in certain dissected limestone islands that are believed to represent uplifted atolls.[1] On the contrary, in the few uplifted atolls which show their foundation rocks, these rocks have an irregular surface, taken to be the product of subaerial erosion, on which the covering limestones rest uncomformably; and this proves the occurrence of subsidence in association with the atoll formation, and previous to its upheaval. One of the best examples of such an uplifted atoll is Tuvutha in eastern Fiji, as described by Foye.[2]

4. On the other hand, if lagoon-enclosing reefs have grown up from subsiding foundations, it is most natural that the enclosed lagoons should

never have an excessive depth; for the very fact that the lagoon floors are reef-enclosed proves that any subsidence which has taken place during their formation cannot have been faster than the slow rate of reef-upgrowth; and the effect of such subsidence in deepening a lagoon may well have been nearly counter-balanced by the effect of inwashed and of locally supplied detritus in shoaling it. It is curious that this simple consideration has not hitherto received more consideration. It should be here noted that the rate of reef upgrowth must be much slower than the upgrowth rate of corals and other organisms on the outer or growing face of a reef; for the organisms that grow there have not only to build up the growing face itself, but also to supply detritus for building up in corresponding proportion the long, steep-pitching talus by which rapid descent is made to deep water, as well as for building up the reef flat and in part for aggrading the enclosed lagoon floor. Hence the rate of subsidence in the case of reef-enclosed lagoons may be only a tenth, more or less, of the rate of coral upgrowth. A maximum depth of 60, 70 or 80 fathoms seems to have been thus imposed on enclosed lagoon floors.

5. The mountain-top islets of almost-atolls afford a rough measure, not heretofore recognized, of the rate of the subsidence by which the lower slopes of the mountain mass was submerged, in terms of the rate of the subaerial degration by which the original volcanic mountain has been sculptured. These islets have such forms as to suggest that their height above the original sea-level base of the island from which they are eroded is about a half of the original height of the island. Hence the subsidence by which the lower half of the original island height has been submerged should be of about the same rate as the degradation by which the upper half of the original height has been removed. Or in more general terms, if $\frac{m}{n}$ of the original height has been disposed of by subsidence when the almost-atoll stage is about to pass into the atoll stage, then subsidence equals $\frac{m.\ \text{degradation}}{n-m}$. In view of so slow a measure of subsidence, it is not surprising that reef-enclosed lagoons are limited in depth to a moderate measure.

6. That the inwash of reef detritus contributes in a significant degree to the aggradation of lagoon floors is conclusively proved by Daly's tables,[3] in which large lagoons are shown to be on the average deeper than small ones: for this manifestly means that the detritus supplied from a given length of exterior reef face has a greater area to aggrade in large lagoons than in small ones. A more direct indication that the inwash of detritus takes place is found in the occurrence of abundant reef blocks and fragments on the reef flat, as well as by the frequent presence of slopes of coral sand, on which occasional reef blocks are strewn, in the lagoon waters next

inside of the reef flat. That the lagoon waves and currents are competent to shift detritus on the lagoon floor at time of gales and storms is proved by the turbidity of the lagoon water on such occasions. That a lagoon floor should become nearly level as it is more and more aggraded, whatever the shape of its foundation, is a necessary result of the lifting of detritus by storm waves chiefly from shoals and of its settling from the turbid water everywhere. The inequalities of depth must be reduced to small measure by the long continued action of that process. The smoothest floors are found in "drowned atolls," where the large waves of the open ocean run over the submerged reef in considerable force.

7. There is probably a spontaneous interaction of several processes by which the effect of an increased rate of subsidence in producing an increase of lagoon depth, or of a decreased rate of subsidence in producing a decrease of lagoon depth, is diminished. For if a reef be slightly and temporarily submerged by increased subsidence, the inwash of detritus from it and the aggradation of the lagoon will be more active and the deepened lagoon will soon be shoaled; but if a reef subside very slowly, its flat will be broadened and inwash and aggradation will be less active; indeed if subsidence is unusually slow, a broadened reef flat may be built up with sand islands, and across such islands there will be no inwash. However, it would seem probable that for a time after a relatively sudden subsidence by which a surface reef is converted into a "sunken barrier," the lagoon behind it will be deeper than usual. The long barrier reef of the southern coast of eastern New Guinea affords two instances of this kind: in a total length of 380 nautical miles, the reef is submerged in two stretches of about 60 and 140 miles; the lagoon varies from 5 to 20 miles in width and is ordinarily from 25 to 30 fathoms in depth; but back of the longer "sunken-barrier" its depth is from 45 to 59 fathoms.[4]

8. In connection with the lowering of the Glacial ocean, there has probably been an effective deepening of any lagoons that may have been exceptionally shallow in Preglacial time; for the floors of such lagoons would have been laid dry when the ocean surface sank below their level, and they would have thus been exposed to degradation, chiefly by the removal of their fine-grained limestones in solution. This process, to which insufficient attention has been given, may not be so effective in developing late-Glacial lagoon floors at a standard depth in the coral seas as low-level abrasion has presumably been in developing platforms of standard depth in the marginal belts; but the process is surely too important to be neglected. Its efficiency may have been greater than would appear on first thought; for while the vehement process of abrasion acts only on a peripheral line and is retarded as it advances, and while ordinary erosion acts only on a surface and is retarded as the surface is lowered, degradation by solution acts through almost the whole volume of the emerged lagoon floor

deposits, and its action is as effective after they are worn down low as when they stood at their original height of emergence.

Indeed, if rainfall be abundant, as it must be in the equatorial rain belt, it would seem that not much more time should be required to remove a thick mass of emerged lagoon-floor limestone than a thin mass, because the application of the solvent agency increases at about the same rate as the volume of limestone to be dissolved. Hence if two shallow Preglacial lagoon floors 16 miles in diameter are laid bare in a glacial epoch, one in the marginal belt, the other in the coral seas, with their surfaces 100 feet above the lowered sea level, the waves of the marginal belt would have to work about 400 times as rapidly as solutional degradation in the coral seas in order that both processes should remove the emerged limestones in the same period of time. If any knobs of limestone remained when normal ocean level was regained, they would serve as supports for lagoon-floor reefs patches, such as are not uncommon today: the occurrence of such reef patches is less easily explained over an abraded rock platform.

9. The moderate depth of known submarine banks in the coral seas is more of a problem, under either of the coral-reef theories here considered, than the depth of reef-enclosed lagoon floors. For in the absence of surface reefs, no limit is set to the rate of subsidence by which such banks may have been submerged. According to the Glacial-control theory, submarine banks should not, insofar as they represent stable islands or reefs truncated by low-level abrasion, have a greater depth than about 40 fathoms; yet some are known to have depths of 60, 70, or 80 fathoms. According to the subsidence theory, some such banks might be expected to lie at least as far below sea level as elevated reefs stand above it; yet no banks of such depths are known. To be sure, the deeper banks, like the deeper reef-enclosed lagoons, can be explained according to the Glacial-control theory by assuming that they have subsided since their postulated rock platforms were abraded and aggraded; but every such assumption weakens the theory. On the other hand, submarine banks with depths of from 60 to 80 fathoms are perfectly expectable according to the subsidence theory; and the absence of deeper banks may be for the present plausibly explained as the result of imperfect exploration: enormous spaces in the Pacific have no deep soundings whatever.

10. When all these elements of the problem are considered together, and especially when due attention is given to the manifest effects of low-level abrasion on the islands of the marginal belt in contrast to the prevailing absence of such effects on the islands of the coral seas, and also to the strong variation of depth in lagoons of similar size, and to the competence of lagoon waves to produce smoothly aggraded lagoon floors, neither the evidence for nor yet the need of low-level abrasion in the coral seas is com-

pelling. On the other hand, the evidence for subsidence of slow rate remains unimpaired.

[1] W. M. Davis, The Structure of high-standing Atolls, these PROCEEDINGS, **3**, 1917 (473–479).

[2] W. G. Foye, Geological Observations in Fiji, *Proc. Amer. Acad. Arts Sci., Boston*, **54**, 1918 (1–145), see p. 56.

[3] R. A. Daly, The Glacial-control Theory of Coral Reefs, *Ibid.*, **51**, 1915 (157–251), see p. 191.

[4] British Admiralty charts 2122, 2123.